Soit le polynome $P(x) = x^3 - (a-b)x^2 + (a-3b-1)x + 2\sqrt{2}$

- 1. Determiner a et b tels que $P(x)$ est divisible par $x-2$ et $x+\sqrt{2}$, càd 2 et $-\sqrt{2}$ sont des racines

On a $\begin{cases} P(2) = 0 \\ P(-\sqrt{2}) = 0 \end{cases}$

éq.à $\begin{cases} 8 - (a-b)\times 4 + (a-3b-1)\times 2 + 2\sqrt{2} = 0 \\ -2\sqrt{2} - (a-b)\times 2 + (a-3b-1)(-\sqrt{2}) + 2\sqrt{2} = 0 \end{cases}$

éq.à $\begin{cases} a + b - 3 - \sqrt{2} = 0 \\ (-2-\sqrt{2})a + (2+3\sqrt{2})b + \sqrt{2} = 0 \end{cases}$

éq.à $\begin{cases} b = 3 + \sqrt{2} - a \\ (-2-\sqrt{2})a + (2+3\sqrt{2})(3+\sqrt{2}-a) + \sqrt{2} = 0 \end{cases}$

éq.à $\begin{cases} b = 3 + \sqrt{2} - a \\ (-2-\sqrt{2}-2-3\sqrt{2})a + 6 + 2\sqrt{2} + 9\sqrt{2} + 6 + \sqrt{2} = 0 \end{cases}$

éq.à $\begin{cases} b = 3 + \sqrt{2} - a \\ (-4-4\sqrt{2})a + 12 + 12\sqrt{2} = 0 \end{cases}$

éq.à $a = \dfrac{12+12\sqrt{2}}{4+4\sqrt{2}} = 3$ et $b = \sqrt{2}$

- 2. On pose $a = 3$; $b = \sqrt{2}$

Donc $P(x) = x^3 - (3-\sqrt{2})x^2 + (2-3\sqrt{2})x + 2\sqrt{2}$

2.1 Pour Determiner $Q(x)$ tel que $P(x) = (x-2)Q(x)$,

On effectue la division euclidienne de $P(x)$ par $x-2$

$$\begin{array}{r|l} x^3 - (3-\sqrt{2})x^2 + (2-3\sqrt{2})x + 2\sqrt{2} & x-2 \\ \cline{2-2} x^3 - 2x^2 \qquad\qquad\qquad\qquad\qquad & x^2 - (1-\sqrt{2})x - \sqrt{2} \\ \hline (-1-\sqrt{2})x^2 + (2-3\sqrt{2})x \qquad & \\ (-1-\sqrt{2})x^2 + (2-2\sqrt{2})x \qquad & \\ \hline -\sqrt{2}x + 2\sqrt{2} & \\ -\sqrt{2}x + 2\sqrt{2} & \\ \hline 0 & \end{array}$$

Donc $Q(x) = x^2 - (1-\sqrt{2})x - \sqrt{2}$

2.2. On a $Q(-\sqrt{2}) = (-\sqrt{2})^2 - (1-\sqrt{2})(-\sqrt{2}) - \sqrt{2} = 0$

2.3. La division euclidienne de $Q(x)$ par $x+\sqrt{2}$:

$$
\begin{array}{r|l}
x^2-(1-\sqrt{2})x-\sqrt{2} & x+\sqrt{2} \\ \cline{2-2}
x^2+\sqrt{2}x & x-1 \\ \cline{1-1}
-x-\sqrt{2} & \\
-x-\sqrt{2} & \\ \cline{1-1}
0 &
\end{array}
$$

Alors $Q(x)=(x-1)(x+\sqrt{2})$ et $P(x)=(x-2)(x-1)(x+\sqrt{2})$

2.4. Résoudre $P(x)<0$:

x	$-\infty$		$-\sqrt{2}$		1		2		$+\infty$
$x-2$		$-$		$-$		$-$	0	$+$	
$x-1$		$-$		$-$	0	$+$		$+$	
$x+\sqrt{2}$		$-$	0	$+$		$+$		$+$	
$P(x)$		$-$	0	$+$	0	$-$	0	$+$	

Donc l'ensemble des solutions de l'inéquation est : $S=]-\infty,-\sqrt{2}[\cup]1;2[$

- 3. Soit $x\in]0,1[$. Encadrer $P(x)=x^3-(3-\sqrt{2})x^2+(2-3\sqrt{2})x+2\sqrt{2}$

On a $0<x<1$ donc $0<x^3<1$ et $0<x^2<1$

$$\text{Donc}\begin{cases} 0<x^3<1 \\ -(3-\sqrt{2})<-(3-\sqrt{2})x^2<0 \\ 2-3\sqrt{2}<(2-3\sqrt{2})x<0 \end{cases}$$

Donc $-1-2\sqrt{2} < x^3-(3-\sqrt{2})x^2+(2-3\sqrt{2})x < 1$

Donc $-1-2\sqrt{2}+2\sqrt{2} < x^3-(3-\sqrt{2})x^2+(2-3\sqrt{2})x+2\sqrt{2} < 1+2\sqrt{2}$

Donc $-1<P(x)<1+2\sqrt{2}$

alors $-1+\sqrt{2}<P(x)-\sqrt{2}<1+\sqrt{2}$

Donc $|P(x)-\sqrt{2}|<1+\sqrt{2}$. la valeur approchée: $\sqrt{2}$, la précision: $1+\sqrt{2}$

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