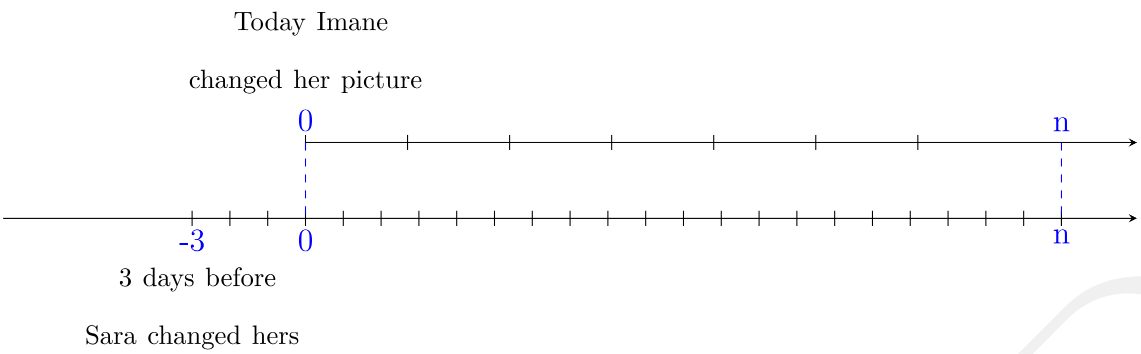
Sara changes her profile picture every 25 days. Her friend Imane changes hers every 31 days. if Imane changed her picture today and Sara changed hers 3 days ago. how many days will pass before they change their profile pictures in the same day?



Let n be the number of days

Let $n\%31 = r_1$ the remaining days after the last profile change made by Imane

Let $(n-3)\%25 = r_2$ the remaining days after the last profile change made by Sara

$n\%31 = r_1 \Leftrightarrow \exists k \in \mathbb{N} \ ; \ n = 31k + r_1$

$(n-3)\%25 = r_2 \Leftrightarrow \exists k' \in \mathbb{N} \ ; \ n - 3 = 25k' + r_2$

$$\Leftrightarrow \begin{cases} r_1 = n - 31k \\ r_2 = n - 3 - 25k' \end{cases} \qquad (1)$$

when they make the change at the same time, the remaining days after both changes are equal

$r_1 = r_2$

thus $n - 31k = n - 3 - 25k'$

$$\Leftrightarrow \boxed{31k - 25k' = 3} \quad (2)$$

Euclid's algorithm:

$$\begin{array}{rcl} 31 & = & 25 \times 1 + 6 \\ 25 & = & 6 \times 4 + \underline{1} \qquad \Rightarrow gcd(31,25) = 1 \\ 6 & = & 6 \times 1 + 0 \end{array}$$

$$\begin{array}{rcl} 1 & = & 25 - 6 \times 4 \\ 1 & = & 25 - (31 - 25 \times 1) \times 4 \\ 1 & = & 31(-4) + 25(5) \\ 3 & = & 31(-12) - 25(-15) \end{array}$$

we substitute 3 in the equation (2) :

$31k - 25k' = 31(-12) - 25(-15)$

$$\Leftrightarrow \begin{cases} 31(k+12) = 25(k'+15) \\ gcd(31,25) = 1 \end{cases} \qquad \Rightarrow 31 | k' + 15$$

$\Rightarrow k' + 15 = 31\alpha$

$\Rightarrow 31(k+12) = 25 \times 31\alpha$

$\Rightarrow k + 12 = 25\alpha$

$\Rightarrow k = 25\alpha - 12 \ \ and \ \ k' = 31\alpha - 15$

with $\alpha = 1$: $\quad k = 13 \ \ and \ \ k' = 16$

At the n^{th} day where the last change occurs.

we have $r_1 = r_2 = 0$

therefore the equation (1) becomes:

$$\begin{cases} n = 31k = 31 \times 13 \\ n = 25k' - 3 = 25 \times 16 + 3 \end{cases}$$

$$\text{finally} \begin{cases} n = 403 \\ n = 403 \end{cases}$$

They will change their profile pictures in the same day after 403 days.

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