OBJECTIF : AUTOMATISMES DÉRIVATION ET LIMITES

Pour chacune des fonctions suivantes, calculer les limites aux bornes de D_f et déterminer la fonction dérivée f'.

1. $e^x - x$
2. xe^x
3. $(x^2-1)e^x$
4. $\frac{e^x}{x}$
5. $e^{2x} - 3e^x + 2$
6. $\frac{1}{e^x+1}$
7. e^{-x^2}
8. $(x+1)e^{-x}$
9. $x - e^x$
10. $\frac{e^x-1}{e^x+1}$
11. $e^{\frac{1}{x}}$
12. x^2e^x
13. $\frac{x}{e^x}$
14. e^{3x-2}
15. $\sqrt{e^x}$
16. $(e^x-2)^2$
17. xe^{-x^2}
18. $\frac{e^x}{x^2+1}$
19. $\ln(e^x+1)$
20. $e^x + e^{-x}$
21. $x^2 - e^x$
22. $e^{2x} - e^x$
23. $\frac{e^x}{x-1}$
24. $(1-x)e^x$
25. $e^{\sqrt{x}}$
26. x^3e^{-x}
27. $\frac{1-e^x}{x}$
28. e^{x^2+x+1}
29. $(x^2+x)e^{-x}$
30. $\frac{e^{2x}}{x}$
31. $e^x \ln(x)$
32. $2e^x - x^2$
33. $\frac{e^x+e^{-x}}{2}$
34. $(x-3)^2e^x$
35. $e^{-x}\sin(x)$
36. $\frac{x^2-1}{e^x}$
37. e^{1-x}
38. $\ln(1+e^{-x})$
39. $e^x + \frac{1}{e^x}$
40. $xe^{1/x}$
41. $e^{\cos x}$
42. $(e^x+1)^3$
43. $\frac{1}{e^x-1}$
44. x^ne^x
45. $e^x - \sqrt{x}$
46. $\frac{x+e^x}{x-e^x}$
47. e^{x^2-4}
48. $(2x+1)e^{2x}$
49. $\frac{e^x}{\sqrt{x}}$
50. $e^x + x^3 - 1$
51. e^{e^x}
52. $x - 1 + e^{-x}$
53. $\frac{e^{2x}-1}{e^x}$
54. $(x^2-x+1)e^x$
55. $\sqrt{1+e^x}$
56. $\frac{x}{1+e^{2x}}$
57. $e^{-1/x}$
58. $x\ln(e^x+1)$
59. $e^{2x} - 4e^x$
60. $\frac{e^x}{x+1}$
61. $e^{x+\ln x}$
62. $(x-1)e^{-x} + 2$
63. $\frac{e^x-x}{e^x}$
64. e^{x^3}
65. $x^2e^{-x} + x$
66. $\frac{e^x}{\ln x}$
67. e^{1/x^2}
68. $(x+1)^2e^{-x}$
69. $\ln(e^x-1)$
70. $\frac{e^x+x}{x^2}$
71. $e^{\tan x}$
72. $e^{2x}\sqrt{x}$
73. $\frac{1-x}{e^x}$
74. $x^x = e^{x\ln x}$
75. $e^x(x-1)^2$
76. $\frac{e^x+2}{e^{2x}}$
77. $e^{-x}\ln(x)$
78. $\sqrt{e^{2x}+1}$
79. $x^2 + e^{-x}$
80. $\frac{\ln(e^x)}{x}$
81. $e^{\sin^2 x}$
82. $(x^3-1)e^x$
83. $\frac{e^x}{x^n}$
84. $e^x + \cos x$
85. $x(e^x-1)$
86. $\frac{x+1}{e^x-1}$
87. $e^{-x} + \sqrt{x}$
88. $x^2e^{1/x}$
89. $\frac{e^x}{1-e^x}$
90. e^{2x-x^2}
91. $\ln|e^x-2|$
92. $x^2e^{-x^2}$
93. $\frac{e^x-e^{-x}}{e^x+e^{-x}}$
94. $(2-x)e^x + x$
95. $e^{\sqrt{x^2+1}}$
96. $\frac{x}{e^x+x}$
97. $e^x(1-e^x)$
98. $\frac{1}{xe^x}$
99. $\ln(e^{2x}+e^x+1)$
100. $xe^x - e^x + 1$
101. $f_{141}(x) = \frac{e^x-e^{-x}}{2}$
102. $f_{142}(x) = \ln(x^2e^x+1)$
103. $f_{143}(x) = \frac{x^2+1}{e^x}$
104. $f_{144}(x) = e^x\sqrt{\ln(x)}$
105. $f_{145}(x) = \ln(1-e^{-x})$
106. $f_{146}(x) = \frac{e^{2x}+3}{e^x-1}$
107. $f_{147}(x) = (x^2-2x+2)e^x$
108. $f_{148}(x) = e^x\cos(e^x)$
109. $f_{149}(x) = \ln(e^x + \sqrt{e^{2x}-1})$
110. $f_{150}(x) = \frac{x}{e^x-x}$
111. $f_{151}(x) = (e^x - x)\ln(e^x-x)$
112. $f_{152}(x) = \sqrt{e^{2x}+e^x+1}$
113. $f_{153}(x) = \frac{1+\ln(x)}{e^x}$
114. $f_{154}(x) = e^{-x}\ln(1+e^x)$
115. $f_{155}(x) = \frac{e^x}{x^2\ln(x)}$
116. $f_{156}(x) = (x+1)^ne^{-x}$
117. $f_{157}(x) = \ln|\frac{e^x-1}{e^x+1}|$
118. $f_{158}(x) = e^{x^2}\ln(x^2+1)$
119. $f_{159}(x) = \frac{xe^x}{x+1}$
120. $f_{160}(x) = \ln(2e^x+3)$
121. $f_{161}(x) = e^{\frac{x-1}{x+1}}$
122. $f_{162}(x) = \frac{\ln(e^x+x)}{x}$
123. $f_{163}(x) = (e^x+x)^2$
124. $f_{164}(x) = \sqrt{x^2+1}e^{-x}$
125. $f_{165}(x) = \ln(1+xe^x)$
126. $f_{166}(x) = \frac{e^{3x}}{x^3+1}$
127. $f_{167}(x) = e^x(x-\ln x)$
128. $f_{168}(x) = \frac{1}{e^x+e^{-x}}$
129. $f_{169}(x) = \ln(x^2e^{-x})$
130. $f_{170}(x) = (x^3-x)e^{1/x}$
131. $f_{171}(x) = \frac{e^x}{\sqrt{x^2+x+1}}$
132. $f_{172}(x) = \ln|\sin(e^x)|$
133. $f_{173}(x) = e^x\tan(e^x)$
134. $f_{174}(x) = \frac{x^2-e^x}{x^2+e^x}$
135. $f_{175}(x) = \ln(e^x+e^{-x})$
136. $f_{176}(x) = e^{\sqrt{\ln x}}$
137. $f_{177}(x) = \frac{\ln(x)}{e^x-1}$
138. $f_{178}(x) = (x-e^x)^n$
139. $f_{179}(x) = \sqrt{\frac{e^x}{e^x+1}}$
140. $f_{180}(x) = \ln(x^2+e^{2x})$
141. $f_{181}(x) = e^{x^2-x}\ln(x)$
142. $f_{182}(x) = \frac{xe^x}{1+e^{2x}}$
143. $f_{183}(x) = \ln(x+e^{-x})$
144. $f_{184}(x) = (1-x^2)e^{-x^2}$
145. $f_{185}(x) = \frac{e^x-\ln x}{e^x+\ln x}$
146. $f_{186}(x) = e^{\sin(x)+\cos(x)}$
147. $f_{187}(x) = \sqrt{e^x+x}$
148. $f_{188}(x) = \ln(1+\sqrt{e^x})$
149. $f_{189}(x) = \frac{x^n}{e^{nx}}$
150. $f_{190}(x) = (x+1)e^{1/(x+1)}$
151. $f_{191}(x) = \ln|\frac{1}{1-e^x}|$
152. $f_{192}(x) = e^x\ln(x^2+1)$
153. $f_{193}(x) = \frac{e^x+e^{-x}}{e^x-e^{-x}}$
154. $f_{194}(x) = \ln(1+x^2e^x)$
155. $f_{195}(x) = e^x(\cos x - \sin x)$
156. $f_{196}(x) = \frac{\sqrt{e^x}}{x+1}$
157. $f_{197}(x) = \ln(e^x+e^{2x})$
158. $f_{198}(x) = (x^2+1)e^{\sqrt{x}}$
159. $f_{199}(x) = \frac{e^x}{x\ln^2(x)}$
160. $f_{200}(x) = \ln(\ln(e^x+1))$

Exemples de Corrections Détaillées

2. $f_2(x) = xe^x$

— **Limites :** $\lim_{x\to-\infty} f_2(x) = 0$ (croissance comparée). $\lim_{x\to+\infty} f_2(x) = +\infty$.

— **Dérivée :** $f_2'(x) = 1 \cdot e^x + x \cdot e^x = e^x(x+1)$.

11. $f_{11}(x) = e^{1/x}$

— **Limites :** $\lim_{x\to0^+} f_{11}(x) = e^{+\infty} = +\infty$. $\lim_{x\to0^-} f_{11}(x) = e^{-\infty} = 0$. $\lim_{x\to\pm\infty} f_{11}(x) = e^0 = 1$.

— **Dérivée :** $f_{11}'(x) = -\frac{1}{x^2}e^{1/x}$.

19. $f_{19}(x) = \ln(e^x + 1)$

— **Limites :** $\lim_{x\to-\infty} \ln(0+1) = 0$. $\lim_{x\to+\infty} \ln(e^x(1+e^{-x})) = \ln(e^x) = +\infty$.

— **Dérivée :** $f_{19}'(x) = \frac{(e^x+1)'}{e^x+1} = \frac{e^x}{e^x+1}$.

100. $f_{100}(x) = xe^x - e^x + 1$

— **Dérivée :** $f_{100}'(x) = (e^x + xe^x) - e^x = xe^x$.

— **Note :** C'est la méthode classique pour trouver la primitive de xe^x.