Exercise 1:

1

$$\begin{aligned}(2+3i)+(5-2i) &= 2+5+(3-2)i\\ &= 7+i\end{aligned}$$

2

$$\begin{aligned}(4-2i)-(3+6i) &= 4-3-2i-6i\\ &= 1-8i\end{aligned}$$

3

$$\begin{aligned}(2+i)(3-2i) &= 2\times 3-2\times 2i+i\times 3-2\times i^2\\ &= 6-4i+3i+2\\ &= 8-i\end{aligned}$$

4

$$\begin{aligned}(3-4i)^2 &= 9-24i-16\\ &= -7-24i\end{aligned}$$

5

$$\begin{aligned}(2+i)^3 &= 8+3\times 4i+3\times 2\times i^2+i^3\\ &= 8+12i-6-i\\ &= 2+11i\end{aligned}$$

6

$$\begin{aligned}\frac{3-4i}{1+i}\times\frac{1-i}{2+3i} &= \frac{(3-4i)(1-i)}{(1+i)(1-i)}\times\frac{(1-i)(2-3i)}{(2+3i)(2-3i)}\\ &= \frac{3-3i-4i+4i^2}{1-i^2}\times\frac{2-3i-2i+3i^2}{4-9i^2}\\ &= \frac{3-7i-4}{1+1}\times\frac{2-5i-3}{4+9}\\ &= \frac{-1-7i}{2}\times\frac{-1-5i}{13}\\ &= \frac{1+5i+7i+35i^2}{26}\\ &= \frac{-34+12i}{26}\\ &= \frac{-34}{26}+\frac{12}{26}i\\ &= \frac{-17}{13}+\frac{6}{13}i\end{aligned}$$

7

$$\begin{aligned}\frac{2+3i}{1-i} &= \frac{(2+3i)(1+i)}{(1-i)(1+i)}\\ &= \frac{2+2i+3i-3}{1+1}\\ &= \frac{-1+5i}{2}\\ &= \frac{-1}{2}+\frac{5}{2}i\end{aligned}$$

Exercise 2:

nombre	conjugué	module	forme trigonométrique.
$a+ib$	$a-ib$	$r=\sqrt{a^2+b^2}$	$\theta=\tan^{-1}\left(\frac{\mathrm{Im}(z)}{\mathrm{Re}(z)}\right)=\tan^{-1}\left(\frac{b}{a}\right)$, $z=r(\cos\theta+i\sin\theta)$
$4+2i$	$4-2i$	$\sqrt{4^2+2^2}=\sqrt{20}$	$\theta=\tan^{-1}\left(\frac{2}{4}\right)$ $z=\sqrt{20}\left(\cos\left(\tan^{-1}(\frac{1}{2})\right)+i\sin\left(\tan^{-1}(\frac{1}{2})\right)\right)$
$1-\sqrt{2}i$	$1+\sqrt{2}i$	$\sqrt{1^2+(-\sqrt{2})^2}=\sqrt{3}$	$\theta=\tan^{-1}\left(-\sqrt{2}\right)$, $z=\sqrt{3}(\frac{1}{\sqrt{3}}+\frac{-\sqrt{2}}{\sqrt{3}}i)$ $z=\sqrt{3}\left(\cos\left(\tan^{-1}(-\sqrt{2})\right)+i\sin\left(\tan^{-1}(-\sqrt{2})\right)\right)$
$(2+2i)^5$	$(2-2i)^5$	$(\sqrt{2^2+2^2})^5$ $=(2\sqrt{2})^5$	$z^n=[r(\cos\theta+i\sin\theta)]^n=r^n(\cos n\theta+i\sin n\theta)$ $z=2\sqrt{2}(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2})=2\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$ $z^n=(2\sqrt{2})^5(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4})$
$1+i\sqrt{3}$	$1-i\sqrt{2}$	$\sqrt{1^2+\sqrt{3}^2}=2$	$z=2(\frac{1}{2}+\frac{\sqrt{3}}{2}i)=2(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})$
$\frac{1+i\sqrt{3}}{(2+2i)^5}$		$\frac{2}{(2\sqrt{2})^5}$	$z=\frac{[r,\theta]}{[r',\theta']}=[\frac{r}{r'},\theta-\theta']=\frac{[2,\frac{\pi}{3}]}{[(2\sqrt{2})^5,\frac{5\pi}{4}]}=[(2\sqrt{2})^5,\frac{\pi}{3}-\frac{5\pi}{4}]$ $=[(2\sqrt{2})^5,\frac{-11\pi}{12}]$

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